ON THE PROBLEM OF CONTINUOUS MODELS OF SUSPENSIONS

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Methods of nonequilibrium thermodynamics are used for constructing a model of two-phase medium (incompressible fluid with particles) with allowance for microrotation and microdeformations in each phase and of chaotic fluctuating motion of particles that is defined by the effective fluctuation temperature. Obtained equations define the effects of intensification of diffusion of suspended particles and admixtures in the fluid phase due to fluctuations. Dependence of the transport coefficient on fluctuation temperature reduces in the limit case to the dependence of the rate of shear intensity.

Theoretical difficulties in the application of the microrheological method in dynamics of concentrated suspensions compel us to turn to general methods of mechanics of continuous medium [1-5]. However the derivation of determining equations by such methods results in excessive generality, as soon as one disregards the contraints imposed by linear nonequilibrium thermodynamics; such constraints are sometimes necessary for taking in the model into account phenomena that exist in reality.

It is shown below that a number of important nonlinear effects can be allowed for in the suspension model without resorting to essentially nonlinear modifications in the formalism of description of irreversible phenomena owing to the transition to multi-velocity continuum and the introduction of internal degrees of freedom.

1. Let us consider a mixture consisting of an incompressible Newtonian fluid carring suspended particles of incompressible material, whose concentration is generally not low. We assume that the motion of volumes of material containing a large number of particles is regular (motion of the mixture as a whole is laminar), but the motion of each individual particle are subjected to random fluctuations, and this leads to fluctuations of the surrounding fluid velocity. Only Brownian fluctuations are present in a quiescent fluid; in an inhomogeneous flow fluctuations are considerably increased owing to the interaction ("collisions") of particles. The pseudo-turbulence generated by this results in the alteration of the transport properties, in particular to intensification of diffusion of suspended particles and admixtures dispersed in the fluid phase.

The total energy of phase α($\alpha = 1$ and $\alpha = 2$ relate, respectively, to suspended particles and the carrier) per unit of mass is defined by formula

$$E^\alpha = 1/2\, u_i{}^\alpha u_i{}^\alpha + U^\alpha \qquad (1.1)$$

where $u_i{}^\alpha$ is the mean-mass velocity, U^α is the internal energy of phase which comprises the energies of thermal motion of molecules and of fluctuations.

Each molecule obviously performs both these random motions; it seems reasonable

to consider the thermal motion on the background of fluctuations. If these motions are considered to be statistically independent, it is possible to introduce for a number of parameters, such as internal energy, entropy, heat flux, stress tensor, etc., additive components which correspond to thermal and fluctuation motions which are denoted below by subscripts T and F

$$U^\alpha = U_T{}^\alpha + U_F{}^\alpha, \quad S^\alpha = S_T{}^\alpha + S_F{}^\alpha, \quad q_i{}^\alpha = q_{Ti}^\alpha + q_{Fi}^\alpha, \quad \ldots$$

The basic assumption used subsequently is that the relation between components U^α and S^α is of the form

$$U^1 = U_T{}^1 (S_T{}^1) + U_F{}^1 (\rho^1, S_F{}^1)$$
$$U^2 = U_T{}^2 (S_T{}^2, c \,/\, C^2) + U_F{}^2 (\rho^2, S_F{}^2)$$

(1.2)

where $\rho^\alpha = \rho C^\alpha$ are mean phase densities, C^α are mass concentrations ($C^1 \equiv C$, $C^2 \equiv 1 - C$), c is the mass concentration of admixture in the fluid phase (per unit mass of mixture), and $\rho = $ const is the density of mixture. The true densities of phases are assumed below to be the same and equal ρ.

For the mixture as a whole

$$E = \Sigma C^\alpha E^\alpha, \quad S = \Sigma C^\alpha S^\alpha, \quad u_i = \Sigma C^\alpha u_i{}^\alpha; \quad V_i{}^\alpha = u_i{}^\alpha - u_i$$

(1.3)

where the symbol Σ denotes here and in what follows summation over α (over phases).

The Gibbs identities specified in the form

$$dU_T{}^1 = T_T{}^1 \, dS_T{}^1, \quad dU_T{}^2 = T_T{}^2 dS_T{}^2 + \mu d \, (c \,/\, C^2)$$
$$dU_F{}^\alpha = T_F{}^\alpha dS_F{}^\alpha + p_F{}^\alpha (\rho^\alpha)^{-2} \, d\rho^\alpha$$

(1.4)

define for each phase the "thermal" and "fluctuation" temperatures $T_T{}^\alpha$ and $T_F{}^\alpha$, the fluctuation pressures $p_F{}^\alpha$, and the admixture chemical potential μ. Generally $T_T{}^1 \neq T_T{}^2$ and $T_F{}^1 \neq T_F{}^2$, however, for simplicity we disregard these inequalities and set $T_T{}^\alpha = T_T$ and $T_F{}^\alpha = T_F$.

In (1.2) and (1.4) functions $U_T{}^\alpha$ play a part similar to that of internal energy in homogeneous incompressible media, while functions $U_F{}^\alpha$ are similar to those defining the internal energy of compressible gas [1].

In the equilibrium thermodynamic state $U^\alpha = U^\alpha (S^\alpha, \rho^\alpha, c)$, hence the Jacobian $D \, (U^\alpha, S^\alpha) \,/\, D \, (S_T{}^\alpha, S_F{}^\alpha)$ vanishes and $T_F = T_T$ (only Brownian fluctuations are present).

Note that the simplest of possible definitions (see Sect. 5 below) of fluctuations by the introduction in the analysis of energy U_F or of the effective temperature T_F was proposed earlier (see, e.g., [4,6,7], but the complete scheme of reasoning peculiar to nonequilibrium thermodynamics did not take these parameters into account.

Formulas (1.2)-(1.4) are used below together with the laws of conservation for obtaining the equation of entropy balance, and from it the determining equation. The analysis of the latter is aimed at pointing out some useful generalizations within the bounds of the traditional phenomenological approach and with a more detailed

description that takes into account the two-phase effect and the fluctuations.

2. We define the mixture kinematics by mean-mass velocities of phases $u_i{}^\alpha$ and tensors $w_{ij}{}^\alpha$ whose symmetric parts $w_{(ij)}^\alpha$ represent rates of deformation and the antisymmetric parts $w_{[ij]}^\alpha$ represent angular velocities of microrotations. Rectangular Cartesian coordinate system is used here and subsequently.

The laws of conservation of the mass of phases and of admixture, of momentum and moment of momentum are of the form

$$\partial_t \rho^\alpha + \partial_l (\rho^\alpha u_l{}^\alpha) = 0 \tag{2.1}$$

$$\rho (\partial_t c + u_l \partial_l c) = -\partial_l j_l \tag{2.2}$$

$$\partial_t (\rho^\alpha u_k{}^\alpha) + \partial_l (\rho^\alpha u_l{}^\alpha u_k{}^\alpha) = -\partial_l p_{lk}^\alpha + R_k{}^\alpha + \rho^\alpha f_k{}^\alpha \tag{2.3}$$

$$\partial_t (\rho^\alpha l_{ik}^\alpha) + \partial_l (\rho^\alpha u_l{}^\alpha l_{ik}^\alpha) = -\partial_l G_{l[ik]}^\alpha + x_i (R_k{}^\alpha + \rho^\alpha f_k{}^\alpha) - $$
$$x_k (R_i{}^\alpha + \rho^\alpha f_i{}^\alpha) + \rho^\alpha \Phi_{[ik]}^\alpha + M_{[ik]}^\alpha \quad (l_{ik}^\alpha = x_i u_k{}^\alpha - x_k u_i{}^\alpha) \tag{2.4}$$

where ∂_t and ∂_l denote differentiation with respect to t and x_l ; j_l is the admixture diffusion flux, p_{ik}^α are partial stress tensors $f_k{}^\alpha$ and Φ_{ik}^α are external mass forces and moments, and $R_k{}^\alpha$ and M_{ik}^α are volume forces and moments of phase interaction.

Setting

$$G_{l[ik]}^\alpha = -2\mu_{l[ik]}^\alpha + x_i p_{lk}^\alpha - x_k p_{li}^\alpha$$

from (2.3) and (2.4) we obtain

$$-p_{[ik]}^\alpha + \partial_l \mu_{l[ik]}^\alpha + {}^1/_2 (\rho^\alpha \Phi_{[ik]}^\alpha + M_{[ik]}^\alpha) = 0$$

or

$$-p_{ik}^\alpha + \partial_l \mu_{lik}^\alpha + {}^1/_2 (\rho^\alpha \Phi_{ik}^\alpha + M_{ik}^\alpha) = m_{ik}^\alpha \tag{2.5}$$

where m_{ik}^α are symmetric tensors of "microstresses".

The method of introduction of supplementary kinematic variables for each phase and the formulation of Eq. (2.5) is similar to that used in [2]. The "microinertial" terms have been omitted in formula (1.1) for energy E^α and in expressions for l_{ik}^α for simplicity; this affects neither the form of determining relationships subsequently derived, nor the final conclusions.

The question of conditions that validate this simplification and of the range of problems for which the introduction of further kinematical details is necessary, is beyound the scope of this paper. It should be, however, noted that many consequence of the allowance for fluctuations remain valid in the case of single-velocity definition of the mixture and without the introduction of variables w_{ik}^α.

We stipulate the equation of conservation of total energy of the form

$$\partial_t \rho^\alpha E^\alpha = -\partial_l (\rho^\alpha u_l{}^\alpha E^\alpha + q_l{}^\alpha + u_k{}^\alpha p_{lk}^\alpha - \overset{\bullet}{w}_{ik}^\alpha \mu_{lik}^\alpha) + \tag{2.6}$$
$$u_k{}^\alpha (\rho^\alpha f_k{}^\alpha + R_k{}^\alpha) + {}^1/_2 w_{ik}^\alpha (\rho^\alpha \Phi_{ik}^\alpha + M_{ik}^\alpha) + W^\alpha = 0$$

where q_l^α is the heat flux and W^α the interphase energy flux. In order to have the combined equations (for the mixture) free of phase interaction characteristics, we set

$$\Sigma R_k{}^\alpha = 0, \quad \Sigma M_{ik}^\alpha = 0, \quad \Sigma (W^\alpha + \tfrac{1}{2} M_{ik}^\alpha w_{ik}^\alpha + R_k{}^\alpha u_k{}^\alpha) = 0 \qquad (2.7)$$

From (2.6) using conventional methods for the heat influx we have the equations

$$\rho^\alpha (\partial_t + u_l{}^\alpha \partial_l) U^\alpha = - \partial_l q_l{}^\alpha - p_{(lk)}^\alpha e_{lk}^\alpha -$$
$$p_{[lk]}^\alpha B_{lk}^\alpha + \varkappa_{lk}^\alpha w_{lk}^\alpha + \mu_{lik}^\alpha \partial_l w_{ik}^\alpha + W^\alpha$$

$$e_{lk}^\alpha = \tfrac{1}{2} (\partial_l u_k{}^\alpha + \partial_k u_l{}^\alpha), \quad \Omega_{lk}^\alpha = \tfrac{1}{2} (\partial_l u_k{}^\alpha - \partial_k u_l{}^\alpha)$$

$$B_{lk}^\alpha = \Omega_{lk}^\alpha - w_{[lk]}^\alpha, \quad \varkappa_{lk}^\alpha = p_{(lk)}^\alpha + m_{lk}^\alpha$$

Taking into consideration the remark about the additivity of heat and fluctuation components (Sect. 1) the above equations can be written as

$$\rho^\alpha (\partial_t + u_l{}^\alpha \partial_l) U_T{}^\alpha = - \partial_l q_{Tl}^\alpha - p_{T(lk)}^\alpha e_{lk}^\alpha - p_{T[lk]}^\alpha B_{lk}^\alpha + \qquad (2.8)$$
$$\varkappa_{Tlk}^\alpha w_{(lk)}^\alpha + \mu_{Tlik}^\alpha \partial_l w_{ik}^\alpha + W_T{}^\alpha + \theta^\alpha$$

$$\rho^\alpha (\partial_t + u_l{}^\alpha \partial_l) U_F{}^\alpha = - \partial_l q_{Fl}^\alpha - p_{F(lk)}^\alpha e_{lk}^\alpha -$$
$$p_{F[lk]}^\alpha B_{lk}^\alpha + \varkappa_{Flk}^\alpha w_{(lk)}^\alpha + \mu_{Flik}^\alpha \partial_l w_{ik}^\alpha + W_F{}^\alpha - \theta^\alpha$$
$$(q_l^\alpha = q_{Tl}^\alpha + q_{Fl}^\alpha, \quad p_{lk}^\alpha = p_{Tlk}^\alpha + p_{Flk}^\alpha, \ldots)$$

Parameter θ^α represents the energy exchange between the "degrees of freedom" (i.e. between thermal and fluctuation motions) in the α-phase.

3. Using (1.2)-(1.4), (2.1), and (2.8), for the entropy balance we obtain an equation of the form

$$\rho \dot{\partial}_t S + \partial_l G_l = R \qquad (3.1)$$
$$G_l = \Sigma (\rho^\alpha S^\alpha u_l{}^\alpha + q_{Tl}^\alpha T_T{}^{-1} + q_{Fl}^\alpha T_F{}^{-1}) - \mu j_l{}'$$
$$j_l{}' = j_l - \rho c V_l{}^2 \quad (V_l{}^2 = u_l{}^2 - u_l)$$

The dissipation function R (see formulas (3.4) and (3.5) below) is the bilinear form formed by generalized thermodynamic fluxes and forces. The latter contain quantities e_{ij}^α and w_{ij}^α related by conditions of incompressibility of the mixture and individual phases

$$e_{ii} = 0, \quad w_{ii}{}^\alpha = 0 \qquad (3.2)$$

the first of which, with allowance for (1.3) may be written as

$$\Sigma C^\alpha e_{ii}{}^\alpha + (u_i{}^1 - u_i{}^2) \partial_i C = 0 \quad (C = C^1) \qquad (3.3)$$

The expressions for thermodynamic fluxes must, in conformity with the three conditions (3.2), be determined to within the three undetermined scalar parameters. Multiplying the two formulas (3.2) and (3.3) by the scalars b^α and A, respectively,

and adding these zero products to R , after rearrangement of terms we obtain

$$R = \sum_{s=1}^{14} \mathbf{Y}^s \mathbf{X}^s \qquad (3.4)$$

where the thermodynamic forces $\mathbf{X}^s$ and fluxes $\mathbf{Y}^s$ are represented by the following quantities:

$$X^1 = T_F - T_T, \quad Y^1 = T_T^{-1}T_F^{-1}\theta \qquad (3.5)$$

$$X_l^2 = \partial_l T_T, \qquad Y_l^2 = -T_T^{-2}(q_{Tl} - \mu j_l')$$

$$X_l^3 = \partial_l T_F, \quad Y_l^3 = -T_F^{-2}q_{Fl}$$

$$X_l^4 = \partial_l \mu, \quad Y_l^4 = -T_T^{-1}j_l'$$

$$X_l^5 = u_l^1 - u_l^2, \quad Y_l^5 = -T_T^{-1}R_{Tl} - T_F^{-1}R_{Fl} + A\partial_l C$$

$$X_{lk}^6 = e_{lk}^1, \quad Y_{lk}^6 = -T_T^{-1}p_{T(lk)}^1 - T_F^{-1}(p_{F(lk)} - p_F^1\delta_{lk}) + AC^1\delta_{lk}$$

$$X_{lk}^8 = B_{lk}^1, \quad Y_{lk}^8 = -T_T^{-1}p_{T[lk]}^1 - T_F^{-1}p_{F[lk]}^1$$

$$X_{lk}^{10} = w_{lk}^1, \quad Y_{lk}^{10} = T_T^{-1}\varkappa_{Tlk}^1 + T_F^{-1}\varkappa_{Flk}^1 + b^1\delta_{lk}$$

$$X_{lk}^{12} = w_{lk}^1 - w_{lk}^2, \quad 2Y_{lk}^{12} = -T_T^{-1}M_{Tlk} - T_F^{-1}M_{Flk}$$

$$X_{llk}^{13} = \partial_l w_{lk}^1, \quad Y_{llk}^{13} = T_T^{-1}\mu_{Tllk}^1 + T_F^{-1}\mu_{Fllk}^1$$

$$(q_{Tl} = q_{Tl}^1 + q_{Tl}^2, \quad \theta = \theta^1 + \theta^2, \dots,$$

$$R_l = R_l^1 = -R_l^2, \quad M_{lk} = M_{lk}^1 = -M_{lk}^2)$$

Tensors X^7, Y^7, X^9, Y^9, X^{11}, Y^{11}, X^{14}, and Y^{14} are obtained by the substitution of the superscript 2 for 1 in expressions for X^6, Y^6, X^8, Y^8, X^{10}, Y^{10}, X^{13}, and Y^{13}.

As usual, we assume the existence between fluxes and forces of the linear relationship

$$\mathbf{Y}^r = \sum_s \mathbf{A}^{r,s}\mathbf{X}^s$$

If we denote components of tensors $\mathbf{Y}^r$, $\mathbf{X}^s$, and $\mathbf{A}^{r,s}$ by $Y_{i\dots p}^r$, $X_{j\dots q}^s$, and $A_{i\dots p, j\dots q}^{r,s}$, respectively, then according to the Onsager-Casimir principle of symmetry $A_{j\dots q, i\dots p}^{s,r}$ $= \pm A_{i\dots p, j\dots q}^{r,s}$. The signs $\pm$ correspond to cases of same and different parities of forces $\mathbf{X}^s$ and $\mathbf{X}^r$ with respect to microscopic velocities.

The tensors $\mathbf{A}^{r,s}$ (up to and including the sixth) depend on the metric tensor $\mathbf{g}$ and, possibly, on other quantities not comprised in the number of forces and fluxes. To such quantities belong C, c, $d_k = \partial_k C$, T_T, and T_F. It is permitted to introduce in one of the tensors the argument T_T and in the others T_F, but not simultaneously both (this would have been equivalent to the inclusion besides the parameter of state T_T or T_F, also, the rest $T_F - T_T$ which is a thermodynamic force).

The general form of tensors which depend on the indicated set of arguments is established on the basis of the theory of tensor functions [1].

Some of the fluxes in (3.5) are of the form $\mathbf{Y}^r = T_T^{-1}\mathbf{Y}_T^r + T_F^{-1}\mathbf{Y}_F^r + \mathbf{Y}_0^r$,

while ultimately we have to determine the quantity $\mathbf{y}^r = \mathbf{Y}_T{}^r + \mathbf{Y}_F{}^r$, We set

$$T_T^{-1}\mathbf{Y}_T{}^r = \sum_s \mathbf{A}_T^{r,s}\mathbf{X}^s - \mathbf{Z}^r - \mathbf{Y}_{0T}^r, \quad T_F^{-1}\mathbf{Y}_F{}^r = \sum_s \mathbf{A}_F^{r,s}\mathbf{X}^s + \mathbf{Z}^r - \mathbf{Y}_{0F}^r \quad (3.6)$$

$$\mathbf{A}^{r,s} = \mathbf{A}_T^{r,s} + \mathbf{A}_F^{r,s}, \quad \mathbf{Y}_0{}^r = \mathbf{Y}_{0T}^r + \mathbf{Y}_{0F}^r$$

where $\mathbf{A}_T^{r,s}$ and $\mathbf{A}_F^{r,s}$ depend on the same arguments as $\mathbf{A}^{r,s}$, and $\mathbf{Z}^r$ may further depend also on $\mathbf{X}^s$ ($\mathbf{Z}^r$ does not contribute to dissipation). From this

$$\mathbf{y}^r = \sum_s \mathbf{a}^{r,s}\mathbf{X}^s + \mathbf{Z}^r (T_F - T_T) - \mathbf{y}_0{}^r \quad (3.7)$$

$$\mathbf{a}^{r,s} = T_T\mathbf{A}_T^{r,s} + T_F\mathbf{A}_F^{r,s}, \quad \mathbf{y}_0{}^r = T_T\mathbf{Y}_{0T}^r + T_F\mathbf{Y}_{0F}^r$$

Tensor $\mathbf{a}^{r,s}$ retains the characteristic symmetry of $\mathbf{A}^{r,s}$, provided it is observed in the decomposition of the latter into components. It will be readily seen that the parts $\mathbf{A}_T^{r,s}$ and $\mathbf{A}_F^{r,s}$ that disturb the symmetry differ only by their signs, hence in $\mathbf{y}^r$ they can be considered as included in $\mathbf{Z}^r$.

4. The determining relationships and final equations are in the general case difficult to survey. We shall, therefore, present only some of their distinctive properties, assuming the mixture to be isotropic in the sense that tensors $\mathbf{a}^{r,s}$ are independent of $\partial_k C$. We shall also omit without further notice a number of cross effects in determining relationships and assume throughout that tensors $\mathbf{Z}^r$ are equal zero.

In conformity with statements at the end of Sectn. 3 the stress tensors $p_{lk}{}^{\alpha}$ and the interaction force R_{k} can be defined as follows:

$$p_{(lk)}^1 = [p_F^1 + aC^1 - a_0^{6,6}e_{ii}^1 - a_0^{6,7}e_{ii}^2 - (T_F - T_T) a^{6,1}]\delta_{lk} - \quad (4.1)$$
$$a^{6,6}e_{lk}^1 - a^{6,7}e_{lk}^2 - a^{6,10} w_{(lk)}^1 - a^{6,11}w_{(lk)}^2$$

$$p_{(lk)}^2 = [p_F^2 + aC^2 - a_0^{7,6}e_{ii}^1 - a_0^{7,7}e_{ii}^2 - (T_F - T_T) a^{7,1}]\delta_{lk} - $$
$$a_0^{7,6}e_{lk}^1 - a^{7,7}e_{lk}^2 - a^{7,10}w_{(lk)}^1 - a^{7,11}w_{(lk)}^2$$

$$p_{[lk]}^1 = -a^{8,8} B_{lk}^1 - a^{8,9}B_{lk}^2, \quad p_{[lk]}^2 = -a^{9,8}B_{lk}^1 - a^{9,9}B_{lk}^2$$

$$R_k = -a^{5,5} (u_k^1 - u_k^2) + a\partial_k C - a^{5,2}\partial_k T_T - a^{5,3}\partial_k T_F$$

$$(a = A_T T_T + A_F T_F, \quad A = A_T + A_F)$$

Let us analyze the case when the difference between velocities u_k, u_k^1 and u_k^2 is small, and can be neglected, except in the expression for R_k. The equations of momenta for the mixture and suspended phase assume, respectively, the forms

$$\rho d_t u_k = -\partial_k [p - a^1(T_F - T_T)] + (\eta_1 + \eta_4) \partial_l\partial_l u_k + \quad (4.2)$$
$$e_{lk}\partial_l\eta_1 + \Omega_{lk}\partial_l\eta_4 + \partial_l W_{lk} + \rho f_k$$

$$W_{lk} = a^{10}w_{(lk)}^1 + a^{11}w_{(lk)}^2 - \eta_4^1 w_{[lk]}^1 - \eta_4^2 w_{[lk]}^2$$

$$\rho C d_t u_k = -\partial_k[p^1 - a^{6,1} (T_F - T_T)] + (\eta_1^1 + \eta_4^1)\partial_l\partial_l u_k + \quad (4.3)$$
$$e_{lk}\partial_l\eta_1^1 + \Omega_{lk}\partial_l\eta_4^1 + \partial_l W_{lk}^1 + \rho C f_k^1 + R_k$$

$$W_{lk}^1 = a^{6,10}w_{(lk)}^1 + a^{6,11}w_{(lk)}^2 - a^{8,8}w_{[lk]}^1 - a^{8,9}w_{[lk]}^2$$

where $d_t = d / dt$

The rheological coefficients are denoted here as follows:

$$\eta_1{}^1 = a^{6,6} + a^{6,7}, \quad \eta_1{}^2 = a^{7,6} + a^{7,7}, \quad \eta_1 = \eta_1{}^1 + \eta_1{}^2$$
$$\eta_4{}^1 = a^{8,8} + a^{8,9}, \quad \eta_4{}^2 = a^{9,8} + a^{9,9}, \quad \eta_4 = \eta_4{}^1 + \eta_4{}^2$$
$$a^1 = a^{6,1} + a^{7,1}, \quad a^{10} = a^{6,10} + a^{7,10} \quad a^{11} = a^{6,11} + a^{7,11}$$

Pressure p is determined by formula

$$p = a + p_F{}^1 + p_F{}^2$$
$$(p^1 = p_F{}^1 + aC^1 = pC^1 - p_F{}^2C^1 + p_F{}^1C^2)$$

Equation (4.3) is used for determining the diffusion flux of suspended particles

$$J_k \equiv \rho C \, (u_k{}^1 - u_k) = \rho C \, (1 - C) \, (u_k{}^1 - u_k{}^2) \tag{4.4}$$

and Eq. (4.2) makes possible the elimination of the Laplacian $\partial_l \partial_l u_k$ from J_k. After some simple operations we obtain

$$
\begin{aligned}
J_k &= \alpha \rho C \, (1 - C) \, \{ (\lambda - C) \, \partial_k p - (1 - C) \, \partial_k p_F{}^1 + C \partial_k p_F{}^2 - \\
&\quad a^{5,2} \partial_k T_T - a^{5,3} \partial_k T_F + \rho \, [(\lambda - C) \, d_l u_k - (\lambda f_k - C f_k{}^1)] - \\
&\quad (T_F - T_T) \, (\lambda \partial_k a^1 - \partial_k a^{6,1}) - (\lambda a^1 - a^{6,1}) \, \partial_k (T_F - T_T) - \\
&\quad e_{lk} \, (\lambda \partial_l \eta_1 - \partial_l \eta_1{}^1) - \Omega_{lk} (\lambda \partial_l \eta_4 - \partial_l \eta_4{}^1) - \\
&\quad \lambda \partial_l W_{lk} + \partial_l W_{lk}{}^1 \}, \quad \alpha = (a^{5,5})^{-1}, \quad \lambda = (\eta_1{}^1 + \eta_4{}^1) \, (\eta_1 + \eta_4)^{-1}
\end{aligned}
\tag{4.5}
$$

In accordance with (1.2) and (1.4) $p_F{}^1$ and $p_F{}^2$ are functions of C and T_F; it is reasonable to assume that the rheological coefficients depend on the same quantities and on T_T. It then follows from (4.5) that:

$$
\begin{aligned}
J_k &= \rho \, \{ k \partial_k p - D_{lk} \partial_l C - \zeta_{Tlk} \partial_l T_T - \zeta_{Flk} \partial_l T_F \} - \\
&\quad \beta^1 \partial_l w_{(lk)}{}^1 - \beta^2 \partial_l w_{(lk)}{}^2 - \gamma^1 \partial_l w_{[lk]}{}^1 - \gamma^2 \partial_l w_{[lk]}{}^2 + \\
&\quad \chi \, [(\lambda - C) \, d_l u_k - (\lambda f_k - C f_k{}^1)]
\end{aligned}
\tag{4.6}
$$

The elementary formulas that express ζ_{Tlk}, β^α, γ^α, and χ in terms of previously determined coefficients are of little interest and are omitted here. For D_{lk}, k and ζ_{Flk} we have the following equalities:

$$
\begin{aligned}
D_{lk} &= D\delta_{lk} + \alpha C \, (1 - C) \left\{ e_{lk} \left(\lambda \frac{\partial \eta_1}{\partial C} - \frac{\partial \eta_1{}^1}{\partial C} \right) + \right. \\
&\quad \left. \Omega_{lk} \left(\lambda \frac{\partial \eta_4}{\partial C} - \frac{\partial \eta_4{}^1}{\partial C} \right) + \beta_*{}^1 w_{(lk)}^1 + \beta_*{}^2 w_{(lk)}^2 + \gamma_*{}^1 w_{[lk]}^1 + \gamma_*{}^2 w_{[lk]}^2 \right\}
\end{aligned}
\tag{4.7}
$$

$$
\begin{aligned}
D &= \alpha C \, (1 - C) \left[(1 - C) \frac{\partial p_F{}^1}{\partial C} - C \frac{\partial p_F{}^2}{\partial C} + (T_F - T_T) \times \right. \\
&\quad \left. \left(\lambda \frac{\partial a^1}{\partial C} - \frac{\partial a^{6,1}}{\partial C} \right) \right]
\end{aligned}
$$

$$k = \alpha C \, (1 - C) \, (\lambda - C) \tag{4.8}$$

$$\zeta_{Flk} = \zeta_F \delta_{lk} + \alpha C \left(1 - C\right) \left\{ e_{lk} \left(\lambda \frac{\partial \eta_1}{\partial T_F} - \frac{\partial \eta_1^{1}}{\partial T_F}\right) + \right.$$

$$\left. \Omega_{lk} \left(\lambda \frac{\partial \eta_4}{\partial T_F} - \frac{\partial \eta_4^{1}}{\partial T_F}\right) + \beta_{**}^{1} w_{(lk)}^{1} + \cdots \right\} \tag{4.9}$$

$$\zeta_F = \alpha C \left(1 - C\right) \left[\left(1 - C\right) \frac{\partial p_F^{1}}{\partial T_F} - C \frac{\partial p_F^{2}}{\partial T_F} + a^{5,3} + \right.$$

$$\left. \left(\lambda a^{1} - a^{6,1}\right) + \left(T_F - T_T\right) \left(\lambda \frac{\partial a^{1}}{\partial T_F} - \frac{\partial a^{6,1}}{\partial T_F}\right)\right]$$

The analysis of formulas (4.6)-(4.8) makes possible the following qualitative assessment of the character of diffusion of suspended particles.

1°. Presence of the fluctuation mechanism intensifies Brownian diffusion. If we assume that the fluctuating particles behave like gas, i.e. $p_F^{1} \sim C T_F$ and that $p_F^{1} + p_F^{2} \sim T_F$, then $D \sim D_b \left(T_F / T_T\right)$, where D_b is the Brownian diffusion coefficient at temperature T_T. For instance, setting $T_F \sim \eta^{\circ} e^{\circ} l^{3}$, where η° and e° are the characteristic viscosity and deformation rate, and l is the dimension of a suspended particle, on dimensional considerations we obtain the linear relation between D and p_F, and e, that agrees with experimental data [8-10].

2°. An analogy exists between barodiffusion and thermodiffusion with coefficients k and ζ_F.

3°. A diffusion flux may be induced by gradients of microrotation velocity and microdeformation rate to the extent that these velocities contribute to tensors of force stresses.

4°. The dependence of transport coefficients on particle concentration and fluctuation intensity results in the effective anizotropy of the diffusion flux [11].

For the admixture diffusion flux we obtain on the same assumptions as above the formula

$$j_l = \rho c V_l^{2} - T_T \left(a^{4,2} \partial_l T_T + a^{4,3} \partial_l T_F + a^{4,4} \partial_l \mu\right) \tag{4.10}$$

Noting that

$$V_l^{2} = u_l^{2} - u_l = - J_l / \rho \left(1 - C\right), \quad \mu = \mu \left(T_T, \, c / \left(1 - C\right)\right)$$

from (4.10) we have

$$j_l = - \frac{c}{1 - C} J_l - T_T \left[\left(a^{4,2} + \frac{\partial \mu}{\partial T_T}\right) \partial_l T_T + a^{4,3} \partial_l T_F + \right.$$

$$\left. a^{4,4} \left(\frac{\partial \mu}{\partial c} \partial_l c + \frac{\partial \mu}{\partial C} \partial_l C\right)\right]$$

Reverting to (4.6) we obtain

$$j_l = - \rho D_s \partial_l c + \frac{\rho c}{1 - C} \left(D_{kl} - D_s \delta_{kl}\right) \partial_k C + \tag{4.11}$$

$$\left(\frac{\rho c}{1 - C} \zeta_{Fkl} - T_T a^{4,3} \delta_{kl}\right) \partial_k T_F + \ldots, \quad D_s = T_T a^{4,4} \frac{\partial \mu}{\partial c}$$

   Thus the presence of large particles can considerably affect the diffusion of adm-
ixture. First, the effective diffusion coefficient depends on T_F and C (for instance
$D_s \to 0$ when $C \to 1$ when particles are impermeable to the admixture); second,
admixture transport together with the carrier fluid is in the opposite direction to the
diffusion flux J_l of suspended particles, and, third, the direct effect of fluctuation
on the admixture transport is possible (when $J_l = 0$), owing to gradient T_F (see,
e. g., [12-17]).
   Note that the relation of J_k and j_k to $\partial_k C$ and $\partial_k c$ defined by formulas (4.6)
and (4.11) does not satisfy the condition of symmetry, i. e. does not reduce to the form

$$J_k = a_{11}\partial_k C + a_{12}\partial_k c + \ldots, \quad j_k = a_{21}\partial_k C + a_{22}\partial_k c + \ldots \qquad (4.12).$$

where a_{12} and a_{21} are interrelated. Moreover, the presence of admixture does not
generally affect the flux J_k (if the transport coefficients are independent of c), which
conforms to the physical picture. These two consequences would not be obtained if the
mixture was represented as a single-phase fluid, since in that case gradients $\partial_k C$ and
$\partial_k c$ would represent thermodynamic forces and J_k and j_k their associated fluxes.
In the model considered here $\partial_k C$ represents a parameter of state.
   Let us now consider the formula that determines the exchange of energy between
the degrees of freedom

$$\Theta = T_T T_F [a^{1,1}(T_F - T_T) + a^{1,6}e_{ii}{}^1 + a^{1,7}e_{ii}{}^2] \approx$$
$$T_T T_F a^{1,1}(T_F - T_T)$$

   We summate the second of Eqs.(2.8) with respect to α on the same assumptions
as made previously and obtain

$$\rho d_t U_F = -\partial_l q_{Fl} + \Phi_F - \Theta \qquad (4.13)$$

where Φ_F denotes the fluctuation energy generation by dissipative processes:

$$\Phi_F = -p_{F(lk)}e_{lk} - p_{F[lk]}B_{lk} + \ldots .$$

   Equation (4.13) is generally used for the determination of T_F . Of special int-
erest is the particular case of that equation that defines a quasi-stationary mode, when
$\Phi_F \approx \Theta$;   it gives then the possibility of directly linking T_F with tensor invari-
ants that define the kinematics of the medium.
   For instance, if microrotations and microdeformations are neglected in Φ_F then
$T_T T_F (T_F - T_T) a^{11} = \eta_F e_{lk}e_{lk}$.   Hence T_F is determined as a function of
T_T and C , and $I_2 = e_{lk}e_{lk}$. Since the transport coefficients may depend
on T_F, the resulting equations actually define a viscous medium with nonlinear prop-
erties due to fluctuations.
   In such medium the diffusion fluxes and, in particular, the migration of suspended
particles across the direction of basic motion can be induced by the nonuniformity of
the rate of shear, since J_k and j_k depend on $\partial_k T_F$.

   5. The above analysis permits a number of generalizations which do not require
a repetition of basic arguments and do not affect the essential qualitative characteris-
tics of the model. Thus, the allowance for the particle moment of momentum red-
uces to the inclusion in the energy E^α and in Eqs. (2.5) of additional terms whose

form is a priori known [2]. It is evident that the omission of cross effects and, in particular, of the dependence of R_k on gradients $w_{ij}{}^\alpha$, μ and j_{i} on $u_k{}^1 - u_k{}^2$, does not alter the general structure of formulas (4.6) and (4.11), and do not make (4.12) symmetric.

Various modifications of determining equations by nonlinear terms that do not contribute to dissipation [1] are possible (some of these have already appeared in (3.6) and (3.7)). Thus it is possible to substitute for expression (4.1) for the friction force the expression

$$R_k = - a^{5,5} (u_k{}^1 - u_k{}^2) + \ldots + N_{[ik]} (u_i{}^1 - u_i{}^2)$$

where $N_{[ik]}$ is an arbitrary antisymmetric tensor.

The most rigid assumption in the developed approach is the introduction of "progressing" fluctuation temperature suggested in [6] as the unique in essence characteristic of fluctuations. In fact, fluctuating rotations and deformations of particles determined by their effective temperatures, as well as fluctuation anizotropy, may occur. These factors and the effect of particular properties of a mixture (phase compressibility, particle elasticity, orientation of these, variation of particle volume) can be taken into account in a phenomenological approach with suitable change of formulas for internal energy (*).

Simpler models with allowance for fluctuations can evidently be obtained, if at the beginning additional kinematic variables are excluded or the motion of individual phases is not separately analyzed. However in that case the law of suspended particle diffusion does, evidently, not define the barodiffusion in an incompressible medium. Equations that correspond to such simplifications can be obtained on the basis of Sects. 1-4.

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